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The Italian School of Algebraic Geometry and Mathematics Teaching: Methods, Teacher Training, and Curricular Reforms in the Early Twentieth Century

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Abstract

In this paper, I will illustrate the reasons which led early twentieth-century Italian geometers—in particular Segre, Castelnuovo, and Enriques—to become so concerned with problems pertaining to mathematics teaching; describe the epistemological vision which inspired them; discuss the various ways in which this commitment manifested itself (school legislation, teacher training, textbooks, university lectures, publications, etc.); and make evident the influence of Klein's ideas and initiatives in education.

The Italian school of algebraic geometry was born in Turin at the end of the nineteenth century, under the guidance of Corrado Segre (1863–1924). It soon brought forth such significant results that it assumed a leading position (*führende Stellung*) on an international level, as F. Meyer and H. Mohrmann wrote in the *Encyklopädie der mathematischen Wissenschaften*.[1] Segre inspired an atmosphere of work characterized by highly prolific, enthusiastic, and frenetic activity, which Guido Castelnuovo, recalling his years in Turin, would refer to as "Turin's geometric orgies." The mathematicians involved were gifted students preparing their degree theses with Segre, such as Gino Fano (1892), Beppo Levi (1896), Alberto Tanturri (1899), Francesco Severi (1900), Giovanni Zeno Giambelli (1901), Alessandro Terracini (1911), and Eugenio Togliatti (1912). A number of newly graduated students from Italy and abroad were also drawn to Turin by Segre's fame. Among these, the most famous were Castelnuovo (1887–1891), Federico

Amodeo (1890–1891), Federigo Enriques (11.1893–1.1894), the English couple William H. Young and Grace Chisholm (1898–1899), Gaetano Scorza (1899–1900), and the American Julian Coolidge (1903–1904).[2]

The noteworthy scientific results obtained by the school have led many to overlook, or at best to attach only secondary importance to, the issues related to the teaching of mathematics which would occupy many of its members, including Segre himself, his academic associate Gino Loria, Scorza, Severi, and above all, his disciples Guido Castelnuovo (1865–1952) and Federigo Enriques (1871–1946).

An examination of the articles and other works by these authors dedicated to problems pertaining to teaching, together with the manuscripts of university lectures and a number of published and unpublished papers, reveals a clearly defined concept of mathematics teaching. It springs, on the one hand, from the Italian geometers' contact with Felix Klein and his important organizational role in transforming mathematics teaching in secondary and higher education and, on the other, from the way in which these authors themselves conceived of advanced scientific research.

The spread in Italy of Klein's ideas on education

It is well known that Felix Klein always combined advanced-level research with a great deal of interest in organizational and didactic problems pertaining to mathematics teaching at both the secondary and the advanced level. [3]

The methodological assumptions which underpinned Klein's concept of mathematics teaching, as they emerge from his writings and from his activities as president of the International Commission on Mathematical Instruction, can essentially be summarized as follows. First, he desired to bridge the gap between secondary and higher education. In particular, he proposed transferring the teaching of analytic geometry and, above all, of differential and integral calculus, to the middle school level, even in those schools which did not concentrate on the sciences. The concept of function would pervade the whole mathematics curriculum: the famous expression "functional thinking" (*funktionales Denken*) was adopted as a slogan for his reform program. Furthermore he favored a genetic teaching method, that is, one that takes account of the origins and evolution of the subject, and he believed that teachers should capture the interest and attention of their pupils by presenting the subject in an intuitive manner. He stressed the importance of showing the applications of algebra to geometry and vice versa. He suggested highlighting the applications of mathematics to all the natural sciences. He believed in looking at the subject from a historical perspective. In addition, he argued that more space should be dedicated to the "mathematics of approximation" (*Approximationsmathematik*), that is, "the exact mathematics of approximate relations." Lastly, he firmly believed that it was crucial that elementary mathematics viewed from an advanced standpoint play a key role in teacher training.

Klein's initiatives to improve mathematics teaching at the secondary and university levels in Germany arrived in Italy through various channels.

First of all, towards the end of the nineteenth century many young Italian mathematicians who were engaged in advanced research frequently attended German universities—in particular Leipzig and Göttingen, where Klein himself taught—in the course of postgraduate program or on study trips. Giuseppe Veronese (1880–1881), Ernesto Pascal (1888–1889), Segre (summer 1891), Fano (1893–1894), and Enriques (1903) were the most noteworthy of these mathematicians. It is no coincidence that they returned to Italy not only with new ideas about methods and areas of research, but also with new perspectives on teaching mathematics.

After meeting Klein in Göttingen in 1891, Segre wrote to his friend Castelnuovo:

> Nobody who has not already been here could imagine what kind of man Klein is and the unique skill with which he has reorganized mathematics teaching at this university. It has made a profound impression on me. And don't forget that I have already been deeply impressed by a number of scholars on this trip![4]

Upon his return to Turin after several months of advanced study at Göttingen, Fano gave a enthusiastic account of Klein's work, in which he also refers to Klein's teacher training seminars, observing that, "We have a great deal to learn from Germany as far as the relationship between secondary and higher education is concerned" (Fano, 1894, p. 181).

Moreover, a number of Klein's writings touching upon mathematics education were translated into Italian even before he became the president of the International Commission on Mathematical Instruction (Rome 1908), as a result of which his ideas spread worldwide. Besides the *Erlanger Programm*, which Fano translated at Segre's request (Klein, 1889), these early translations included the *Vorträge über Ausgewählte Fragen der Elementargeometrie,*[5] which was translated by Francesco Giudice at the request of Loria, and the lecture *Über Arithmetisierung der Mathematik*, which appeared in the proceedings of the *Circolo Matematico di Palermo*, translated by Salvatore Pincherle (Klein, 1896b).

Klein himself visited Italy first in 1878[6] and again in 1899, stopping over in Florence, Bologna, Rome, and Padua and meeting, among others, Enriques, Castelnuovo, Cremona, Veronese, and Fano.[7]

Finally the as yet unpublished correspondence between Klein and the Italian geometers, Segre, Fano, Loria, Enriques, and Castelnuovo,[8] bears witness to the twofold influence of Klein in Italy in scientific research and in mathematics teaching.

Klein's ideas on mathematics teaching were particularly appreciated by the members of the Italian school of algebraic geometry: a shared approach to scientific research and his "tendency to consider the objects to be studied in the light of visual intuition" (Enriques, 1923, p. 55) brought Klein and the Italian geometers closer together intellectually.

A number of members of Giuseppe Peano's school of mathematical logic also supported certain aspects of Klein's reform program, even though they assigned a central role to logical rigor. Among these were Rodolfo Bettazzi, the founder of the Mathesis Association (the Italian association of mathematics teachers), and Giovanni Vailati, who sought to implement a number of Klein's ideas in a school reform project (cf. Giacardi, 2009).

"Teach to discover": Segre and teacher training

In addition to his courses of advanced geometry, Corrado Segre also ran a course for future teachers in the *Scuola di Magistero* (Teacher Training School) of the University of Turin for nineteen years (from 1887–1888 to 1891–1892 and from 1907–1908 to 1920–1921).

His handwritten lesson notes—in particular *Lezioni di Geometria non euclidea* (1902–1903), *Vedute superiori sulla geometria elementare* (1916–1917); [*Appunti relativi alle lezioni tenute per la Scuola di Magistero*[9]]—and archival documents clearly show the threefold approach of Segre's lessons at the Teacher Training School: theory, methodology, and practice. He took up anew the themes of elementary mathematics studied at the secondary level, making evident from time to time the connections to higher mathematics; he also examined questions of methodology and didactics. Then, in the laboratories-classes, students were taught to impart clear lessons, documented and stimulating.

In the notebook [*Appunti relativi alle lezioni tenute per la Scuola di Magistero*], after beginning with some considerations on the nature of mathematics, the objectives of teaching, and the importance of intuition and rigor, Segre provides future teachers with some methodological instructions which are on the one hand closely related to his particular way of approaching research, and on the other, are the fruit of his own teaching experience and of an attentive examination of legislative measures in various European countries and of educational issues debated at the time.

For Segre, mathematics should teach how "to reason well; not to be satisfied with empty words; to draw conclusions from the hypothesis, to reflect and discover on one's own; … to speak precisely" (p. 42).[10] Teaching at the secondary level should not be considered an end in itself: "it must arise from the external world and then be applied to it" (p. 15); the first approach to mathematics must therefore be experimental and intuitive, so that the student learns "not only to demonstrate truths already known, but to make *discoveries* as well, to solve the *problems* on his own" (p. 16), while "perfect rigor in certain things can be achieved at a later time" (pp. 25–26). Segre was also convinced that presenting some applications to other sciences, such as physics, astronomy, political economy, actuarial mathematics, and geography, can also help render the material more interesting and motivate study.

Thus for Segre, the principal objective of teaching is the development of the powers of reasoning but equally those of intuition; it is not by chance that, as regards the method to be used, his preferences lie with the *heuristic* for presenting the subject, *analytic* for the proofs, and *genetic* for the development of theories. The first, the Socratic method, permits the student to discover mathematical truths on his own; the second allows him to enter into the mathematics "workshop" and understand the "why" of each step in a proof; the third, developing a theory according to the way in which it is formed, represents a good guide to scientific research. Segre also underlines the importance of varying the methods, and above all, of choosing them according to the subject, the pupils, and the time available.

In addition to considerations of a methodological nature, Segre does not hesitate now and then to offer future teachers various bits of practical advice that show how aware he was of students' errors, bad habits, weak points, and idiosyncrasies:

> Avoid being boring! (p. 24);
> Try to stimulate the activity of the student's mind (pp. 26–27);
> Sometimes satisfy the request for a proof which wouldn't be given, but which a more intelligent youngster can understand (p. 27);
> Vary the notations and figures. It shouldn't happen that a youngster does not know how to solve an equation only because the unknown is not called x. Or a geometric proof because the arrangement of the figure has changed (p. 28);
> The calculations should not be too long, because there is no reason to try the patience of children (p. 32);
> Prepare the lesson perfectly … Don't dictate: use a textbook … Be patient with the students; repeat if they have not understood; don't be aghast at errors; try to persuade the students … that they needn't have a gift for mathematics (p. 42).

In the ample bibliography at the end of the notebook, Segre not only offers a very detailed outline of literature related to the problems of teaching the various branches of mathematics, of manuals being used, of workbooks, of books of amusing mathematics, and of history of mathematics, but also shows that he was aware of the legislative measures of various countries, of writings on the foundations of mathematics, and on pedagogy. Segre's principal points of reference were C. A. Laisant, E. Borel, J. Hadamard and H. Poincarè in France, and P. Treutlein, M. Simon, and Klein in Germany, mathematicians who were all involved in improving the role of intuition as opposed to a teaching too marked by logical rigor. In particular, he made Klein's pedagogical assumptions his own: he believed that teaching should be an active process and develop the students' capacity to discover things for themselves; he sought to bridge the gap between mathematics and all natural sciences, wishing to make science teaching more interesting and more in touch with the real world; he held that logical reasoning and intuition are two inseparable aspects of the same process, and it is therefore necessary for teachers to find the correct balance between the two, moving by degrees from the concrete to the abstract; he maintained that the concepts of function and transformation should be introduced at an early level. He also believed in looking at the subject from a historical perspective. Finally, he was absolutely convinced that elementary mathematics viewed from an advanced standpoint should have a key role in teacher training (for more details cf. Giacardi, 2003a).

Although Segre's contribution to the field of mathematics education remained limited to the lessons at the Teacher Training School, he nevertheless left a profound legacy that his disciples, particularly Castelnuovo and Enriques, instilled in their activities as presidents of the Mathesis Association, in their articles and lectures on the problems of education, in significant publications, and in manuals for secondary schools as well.

Guido Castelnuovo and the introduction of *functional thinking* into secondary schools

During the International Congress of Mathematicians held in Rome from April 6 to 11, 1908, a great deal of attention was paid to the various syllabi of mathematics and to the teaching methods in the countries represented.[11] At the suggestion of David Eugene Smith a committee dedicated to issues pertaining to mathematics teaching was created: the International Commission on Mathematical Instruction (ICMI) (cf. Furinghetti, Giacardi, 2008), with Klein as its first president and Castelnuovo, Enriques, and Vailati as the first Italian delegates.

As a delegate and, later, first as a member of the Central Committee of the ICMI and then as vice president, Guido Castelnuovo built up an international network and promoted the exchange of information on new movements for reform in Europe. He took a particular interest in the reform proposed by Klein, whose methodological approach he wholeheartedly endorsed. Castelnuovo's commitment to education manifested itself in various forms: in his activities in the ICMI and the Mathesis Association, of which he was president from 1911 to 1914, in the courses he taught at university, some of which were devoted to teacher training, in the various articles he dedicated to issues relating to mathematics teaching, and in the syllabi he designed for secondary education.

His interest in educational issues arose from social concerns (Castelnuovo, 1914, p. 191). His approach grew out of a lucid critique of the Italian school system, and in particular, of the teaching of mathematics, which in his opinion was too abstract and theoretical: all reference to practical application was neglected and excessive specialization and unnecessary compartmentalization of different areas led to a distorted cultural perspective.

In order to define exactly what secondary schools should be offering to young people, Castelnuovo asked himself the following three questions: Who is middle school education aimed at?; What should the ultimate goal of schooling be?; What skills should teaching develop? He believed that schools should cater above all to young people aiming to go into one of the so-called "free" professions (engineers, doctors, lawyers, etc.) "both because they constitute the majority of school pupils and because the progressive development of the country will rest mainly on their shoulders" (Castelnuovo, 1913, pp. 18–19). The primary aim of middle schools should be to shape members of society because "a school cannot be said to be truly effective if it is not aimed at average levels of intelligence and if it is unable to create that refined democracy which forms the basis of every modern nation" (Castelnuovo, 1909, p. 4). The qualities which teachers must foster and cultivate in their pupils are the creative imagination, the spirit of observation, and the logical faculties. Excessive rigor is to be avoided:

> Middle schools should not furnish their pupils with knowledge, so much as with a desire and a need for knowledge. They cannot seek to provide an encyclopedic knowledge of everything, but can only offer a clear, although necessarily very limited, idea of the principle questions of the various branches of knowledge, and of some of the methods which have been employed in tackling them. […] Of course, this kind of teaching will not be sufficient to provide middle school students with preparation specific to one or another of the faculties of the university. However, this is not the aim of middle schools. They serve simply to provide students with the aptitude to move on to more advanced studies (Castelnuovo, 1910, p. 39).

In the article *Il valore didattico della matematica e della fisica*, which is virtually a manifesto of Castelnuovo's thinking on education, the placing of mathematics and physics side by side is by no means coincidental. Here, in fact, Castelnuovo emphasizes the importance of observation and experiment, the usefulness of constantly confronting abstraction with reality and the importance of practical application as a means of "shedding light on the value of science." Furthermore, he claims that heuristic procedures should be favored for two reasons: "the first, and the most important reason, is that this type of reasoning is the best way to attain to truth, not just in experimental sciences, but also in mathematics itself"; the second is that it is "the only kind of logical procedure that is applicable in everyday life and in all the knowledge involved with it" (Castelnuovo, 1907, p. 336). He concludes his article by recommending that teachers draw on the history of science so that young people understand the relative and provisional nature of every theory.

In 1911, the minister Luigi Credaro set up a *liceo moderno* that differed from the *liceo classico* from the second class onward. The new curriculum replaced Greek with a modern language (German or English), dedicated more attention to the scientific subjects and added elements of economics and law. Castelnuovo drew up the mathematics syllabus,[12] putting a number of Klein's proposals into practice by introducing the notion of function and the concepts of derivative and integral, attaching a greater importance to numerical approximations and coordinating mathematics and physics teaching. Thus the pupils, Castelnuovo says, "will acquire a more correct and balanced idea of the exact sciences nowadays […], find mathematics to be, instead of a logical drudge, a set of tools and results which can be easily applied to concrete problems."[13]

On that occasion Castelnuovo wrote to Klein:

> As far as teaching is concerned, I am sure that you will be happy to hear that the (modern) programs for mathematics that I have had adopted in modern *licei* have been so well received that the Minister for Public Instruction is thinking of introducing them in the classical *licei* and the technical institutes as well, further broadening in these last the program for infinitesimal calculus (Castelnuovo to Klein, 10 March 1915, UBG, *F. Klein* 51).

Castelnuovo was also influenced by Klein in his views on teacher training. In 1909, during the Congress of the Mathesis Association in Padua he explicitly proposed following the example of Göttingen: "At Klein's suggestion, during the spring holidays a number of German universities hold short courses for Middle school teachers. Couldn't we too *set up similar courses in our universities*?" (Castelnuovo, 1909, p. 4).

Like Segre, Castelnuovo introduced a number of topics designed specifically for the cultural training of future mathematics teachers into his geometry courses at the University of Rome. The following modules, into which he inserted methodological considerations, are of particular relevance: *Geometria non-euclidea* (1910–1911), *Matematica di precisione e matematica di approssimazione* (1913–1914), *Indirizzi geometrici* (1915–1916), *Equazioni algebriche* (1918–1919), and *Geometria non-euclidea* (1919–1920) (cf. Gario, 2001–2003).

For example, while treating non-Euclidean geometry in the 1910–1911 course, he underlines the importance of this theme in teacher training, as long as the "various methods by which non-Euclidean geometry was studied (elementary, differential, projective, group-theoretical)" are examined, and its "philosophical interest, both from the logical point of view (with regard to the independence and compatibility of the postulates) and from the physical perspective (i.e., the origins of the postulates and the nature of space) is emphasized" (pp. 2–3).

In the introduction to the 1913–1914 course on the relationship between precise and approximate mathematics, after indicating the various ways in which future teachers can be trained, Castelnuovo quotes Klein:

> The educational value of mathematics would be much enriched if, in addition to the logical procedures needed to deduce theorems from postulates, teachers included brief digressions on how these postulates derive from observation and indicated the coefficients with which theoretical results are verified in real experience … The relationship between problems pertaining to pure mathematics and those pertaining to applied mathematics is very interesting and instructive. Klein, who dedicated a series of lectures to the subject (1901), describes the first of these as problems of "precise mathematics" and the second as problems of "approximate mathematics". In this course we will … more or less follow the general outline of Klein's course. Klein also had another reason for pursuing this line of enquiry, which was his desire to bridge the gap between mathematicians engaged in pure research and those who have to solve problems relating to applied mathematics (pp. 2–3).

It was no coincidence that when ill health prevented Klein from attending the international ICMI conference in Paris in 1914, he asked Castelnuovo to present the opening address in his place (Castelnuovo to Klein, 3 March 1914, UBG, *F. Klein* 51).

Federigo Enriques and scientific "humanitas"

For almost fifteen years starting from 1892, Castelnuovo was Federigo Enriques's scientific guide and mentor. The fellowship between these two mathematicians led to the publication of a number of important works on algebraic surfaces. Enriques and Castelnuovo also shared a profound interest in educational issues and a set of methodological tenets about mathematics teaching. If for Castelnuovo this interest sprang from social concerns, for Enriques it was rooted in his profound philosophical, historical, and interdisciplinary interests and his studies on the foundations of geometry.

In early 1896, Enriques began to study the origins of the postulates of geometry, taking psychological and physiological studies as his starting point. On the basis of research carried out by H. Helmholtz, E. Hering, E. Mach, and above all the German physiologist W. Wundt, Enriques came to the conclusion that the three branches of geometry, namely topology, metric geometry, and projective geometry, were linked to three different kinds of sensations: the general tactile-muscular, those of the "special sense of touch,"[14] and those of vision.[15]

Klein, too, had considered this matter and discussed with Enriques the psychological problems connected with mathematics during his second trip to Italy:

> But the problem we discussed at greatest length was that regarding the psychological issues relating to mathematics. Yesterday morning, as he took leave of me, he said, "We must take up our conversation on these subjects again. I will not forget it."[16]

In the same period, Klein invited Enriques to write a chapter on the foundations of geometry for the *Encyklopädie der mathematischen Wissenschaften*. This was the principal theme discussed during Enriques's stay in Göttingen in 1903:

> As far as my conversation with Klein goes, you already know how interesting it was. In addition to talking about the foundations of geometry, we discussed educational issues at length, and in just a few hours I learned a great deal from him about a lot of things I knew nothing about—specifically about the way in which mathematics teaching is developing in England and Germany.[17]

It was also as a result of Klein's interest that a German translation of Enriques's *Lezioni di geometria proiettiva (Vorlesungen über projective Geometrie)* came out in 1903 (2nd ed. 1915). In his introduction to this book, Klein expresses particular appreciation for Enriques's treatment of the subject, which "is always intuitive, but thoroughly rigorous," and underlines the impact of this kind of research on didactics, writing:

> Italian researchers are also well ahead of us from a practical point of view. They have by no means disdained exploring the educational consequences of their investigations. The high quality textbooks for secondary schools which came out from this exploration could be made available to a broader audience through good translations. And it would seem particularly desirable in Germany when we consider that our own textbooks are completely out of touch with active research (Klein, 1903, p. III).

The idea of the psychological origins of geometrical foundations is just one of the philosophical issues which Enriques meditated upon during his university studies in Pisa, and which would continue to alternate with his mathematical and philosophical research throughout his life. Broad, rich, and sometimes contradictory, it is impossible within the limits of this paper to outline the epistemological vision on which all of Enriques's scientific work was founded, so I will confine myself to indicating the most important factors which inspired his commitment to mathematics education.

First of all, Enriques held a dynamic and genetic view of the scientific process, which he described as a "process that is at once inductive and deductive, which ascends from specific observations to abstract concepts, only to descend again to practical experience. It is a process of continuous development, which establishes a generative relationship between theories and perceives in their succession only an approximation to truth" (Enriques, 1912a, p. 132). For this reason, Enriques criticizes the tendency to present a mathematical theory in a strictly deductive manner at school, as in this way it appears something closed and already perfect, leaving no room for further discovery. Instead, teachers should approach problems with a number of different methods, paying attention to the errors (cf. Enriques, 1942) which have allowed science to move forward, and indicating open questions and new fields of discovery.

These views are connected to Enriques's conception of the nature of mathematical research—typical of the Italian school of algebraic geometry—as something aiming above all at discovery and particularly emphasizing the inductive aspects and intuition: "The main thing is to discover. … A posteriori it will always be possible to give a demonstration," which, "translating the intuition of the discoverer into logical terms, will provide everyone with the means to recognize and verify the truth" (Enriques, Chisini, 1918, II, pp. 307, 318). This belief is naturally reflected in the style of teaching, which should, according to Enriques, take into account the inductive as well as the rational aspect of theories. Logic and intuition are not two distinct faculties of the human intellect; rather, they represent two inextricable aspects of the same process. Teachers should therefore find the right balance between the two. The important thing is to distinguish clearly between empirical observation and intuition on the one hand, and logic on the other. On this subject, Enriques distinguishes between what he calls "small scale logic," the refined and almost microscopically accurate analysis of thought, and "large scale logic," which considers the organic connections in science. He maintains that teaching should above all take "large scale logic" into account, gradually preparing young people to develop a more refined and rigorous approach (Enriques, 1921b, pp. 9–11).

Enriques believed that scientific developments can only be fully understood in their historical connections: "A dynamic vision of science leads us naturally into the territory of history […] history becomes an integral part of science."(Enriques, Chisini, 1915, I, p. XI). Therefore, at school, the origins of each doctrine should be studied, together with its relationships and developments. No theory should be presented as static (Enriques, 1921b, p. 16).

For Enriques, science is the "conquest and activity of the spirit," so it is vital to establish links between scientific knowledge and other intellectual activities.[18] One of Enriques's main strengths is his understanding of the grave danger that cultural isolation poses to science. He always emphasized the importance of "cultivating one's own field of study as a segment of the greater body of science!"(Enriques, 1912b, p. 35). This conviction found expression in Enriques's constant efforts to bridge the gap between mathematics and other scientific and scholarly fields, such as physics, biology, psychology, physiology, philosophy, and history, to attain a unitary vision of culture. By overcoming narrow specialization, the sciences, and especially mathematics, could realize their true humanistic and formative value.

Enriques's conviction that the school system should be responsible for transmitting the unitary nature of knowledge led him to undertake numerous initiatives and assume various institutional roles with a view to improving mathematics teaching. He was president of the Mathesis Association for many years (1919–1932), directed the *Periodico di matematiche* (1921–1938, 1946),[19] a journal aimed specifically at teachers, wrote secondary school textbooks, founded the National Institute for the History of Sciences (1923), and fostered a number of important and successful publishing initiatives.

Klein's example led him to launch, in collaboration with a number of friends and followers, a series of monographs on problems of elementary mathematics from an advanced standpoint. Enriques writes:

> There is no hiatus or gap between elementary and higher mathematics because the latter develops from the former, as the tree grows from the seedling. And, by looking at the tree, we can discover new features of the seedling and understand characteristics whose meaning had previously escaped our understanding. Just so, the development of mathematical problems will throw light on the elementary theories in which they have their roots (Enriques, 1921b, pp. 15–16).

In 1900 these monographs were collected in the *Questioni riguardanti la geometria elementare*; in the preface, Enriques himself wrote, "These topics have recently been developed in a series of conferences held by Mr. Klein, to whom we are at least partially indebted for the idea of writing this volume" (Enriques, 1900, p. VII). This collection was augmented and enriched in successive editions under the new title *Questioni riguardanti le matematiche elementari* (2nd ed. 1912–1914, 3rd ed. 1924–1927). It was translated wholly or in part into German, Russian, Spanish, and French. The themes considered are essentially those dealt with by Klein, who is cited on a number of occasions.

The *Questioni,* as Enriques himself writes, constitute the scientific and methodological basis for the famous elementary geometry textbook he wrote with Ugo Amaldi (1875–1957). This book was gradually improved and perfected over a series of editions.[20] It was adapted to suit various kinds of schools, and is now considered a classic. Here, too, Enriques acknowledged his debt to Klein:

> I am sending you a copy of the 2nd edition of my *Elementi di geometria*. You will recognize the influence of your own ideas and our conversations in Göttingen on the method [I follow in my textbook] which, while remaining rational, lays emphasis on the inductive aspects (Enriques to Klein, 10.1.1905, UBG, *F. Klein* 34).

Among the textbooks that Klein mentions in his essay on geometry teaching in Italy, he refers to the *Enriques–Amaldi,* which he praises for having taken didactic requirements into consideration, thus reconciling logical rigor and intuition (Klein, 1925–1933, II, pp. 245–250).

The methodological vision which underpins the book is, without a doubt, that of Enriques. The preface to the textbook opens with a clear indication of the method its two authors will follow:

> An elementary geometry textbook must satisfy two sets of needs: the scientific and the didactic. A wrong idea of scientific rigor makes some mathematicians believe that the ideal of the science of geometry consists in a systematic exclusion of intuition. According to this premise one would arrive at an abstruse treatment of the elements which would be inaccessible to a beginner and irreconcilable with the educational purpose of geometry. Geometry is a science of observation and reasoning. It should educate young people in both of these faculties. Scientific rigor, as we understand it, has a formative value because it accustoms students to distinguishing between the activity of one faculty and that of the other (Enriques, Amaldi, 1903, p. 1).

The subject is examined using a "rational-inductive" method, with the aim of avoiding the shortcomings typical of Euclidean-style exposition, which by "presenting propositions which are analysed at length in their logical connections and coordinated in a deductive system, hides the process of discovery under a rigidly dogmatic framework" (Enriques 1912b, p. 24). Problems are addressed as follows: beginning with a series of observations, the authors enunciate certain postulates from which the theorems depending on them are developed by logical reasoning; from these theorems, however, they continually turn back to observations or intuitive explanations. Enriques writes, "This approach to the subject gives pupil's minds a clearer vision of how geometrical ideas have been formed. It leads to *demonstrations* and *definitions* which can well be defined as *inductive.*"[21]

The *Questioni riguardanti la Geometria elementare* was written for the *Scuola di Magistero* and was therefore specifically designed for teacher training purposes. Enriques's ideas on teacher training from both the scientific and didactic points of view are clearly expressed in a speech he gave during the fifth Congress of the National Federation of Middle School Teachers, held in Bologna in 1906. On that occasion, Enriques proposed a university course of study that would lead to a *laurea pedagogica* (didactic degree), distinct from the *laurea scientifica* (scientific degree). This course would be divided up into two biennial parts, the first dedicated to acquiring a basic knowledge of the subject, and the second, which would be held at the *Scuola di Magistero*, consisting in "1) courses in those branches of science which are connected to a deeper vision of its elements, 2) … [lessons] on concrete issues pertaining to pedagogy 3) hands-on activities including a period of practical training, both at the university and in a secondary school" (Enriques, 1907, p. 78).

Enriques's proposed solution to the problem of teacher training was immediately criticized by the neo-idealist philosopher Giovanni Gentile who, identifying "knowing" with "knowing how to teach," believed that teacher training consisted in nothing more than "true, deep and sincere scientific training" (Gentile, 1907, pp. 178–179). Thus began the long and complex relationship between these two influential exponents of Italian cultural life in the first half of the twentieth century, which was to bring them into conflict over both philosophy and school reform.

Enriques reiterated his convictions in various writings on mathematics teaching and, in particular, in the open letter to readers which launched the fourth series of the *Periodico di matematiche* (1921) and in the article *Insegnamento dinamico,* which opened the new series.[22] Here Enriques presents a veritable program of activities, based on the following key principles regarding what teachers should do: explain the science that they are teaching in depth and from various points of view, so that it can be mastered from new and higher points of view; use the history of the science in order to attain not so much an erudite knowledge as a dynamic consideration of concepts and theories, through which the unity of thought can be recognized; and bring out the interrelationships between mathematics and the other sciences, physics in particular, in order to offer a broader vision of science and of the aims and meanings of the many different kinds of research.

Another important publishing initiative was set up by Enriques both for teacher training and for achieving his ideal of the *humanitas scientifica*: the series *Per la storia e la filosofia delle matematiche*, which he launched in 1925 with a volume he himself had edited, *Gli Elementi d'Euclide e la critica antica e moderna, (Libri I–IV)* and which arrived at twelve volumes by 1938. The idea for the series had been suggested to him "from practice in the Scuola di Magistero" (Enriques, 1925, p. 7). It was intended for a readership of educators, but was also aimed at students at the secondary level and educated people in general. In selecting topics he particularly favored translations with commentaries, often accompanied with historical notes, of works by important authors of the past (Euclid, Archimedes, Bombelli, Newton, Dedekind, etc.) which might be of relevance to mathematics teaching.

In 1923, Giovanni Gentile, then minister for public education in Italy's Fascist government, carried out a full and organic reform of the school system[23] in accordance with the pedagogical and philosophical theories he had been developing since the beginning of the 1900s. He divided the secondary school system into two branches. The classical-humanistic branch, designed for the ruling classes, was considered absolutely superior to the technical-scientific branch, which, moreover, made access possible to only a limited number of university faculties. The principles of Fascism and the neo-idealist ideology were opposed to the widespread diffusion of scientific culture and, above all, to its interaction with other cultural sectors. Humanistic disciplines were to form the main cultural axis of national life and, in particular, of education. This point of view was, of course, opposed to the *humanitas scientifica* to which Enriques aspired. As president of the Mathesis Association, he engaged in intense negotiation with Gentile, both before and after the law on secondary education was enacted, in the hope of avoiding the devaluation of science teaching. However, the pleas of the Mathesis fell on deaf ears. Unlike Vito Volterra and Castelnuovo, who were in absolute opposition to the Gentile Reform, Enriques assumed and maintained a conciliatory position (cf. for example Enriques, 1929). His ideal was to achieve a fusion between "scientific knowledge" and "humanistic idealism" in a "superior awareness of the universality of thought" (Enriques, 1924, p. 4).

The imbalance between classical and scientific education that was consolidated by the Gentile Reform was destined to last right up to the end of the twentieth century. However, the contributions which mathematicians like Segre, Castelnuovo, and Enriques made to the study of problems relating to mathematics teaching sowed the seeds which, on the one hand, led to didactics becoming a science in its own right in Italy;[24] on the other hand, they drew attention to the much more far-reaching problem of the relationship between the humanistic and scientific culture, which in the mid-twentieth century in Italy and elsewhere, had given rise to the debate about the so-called "two cultures," a debate which is ongoing today.

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Notes

[1] *Encyklopädie der mathematischen Wissenschaften.* III.I1, *Geometrie (*1907–1910). Leipzig: Teubner, pp. V–XI, at p. VI.

[2] The dates between parentheses indicate respectively when the degree was obtained and period of residence in Turin. Cf. Giacardi, 2001 and 2002.

[3] Regarding Klein and his movement to reform mathematics teaching, cf. Rowe, 1985; Schubring, 1989; Nastasi, 2000; Gario, 2006.

[4] Segre to Castelnuovo, 30 June 1891, in Gario & Palleschi, 1998.

[5] Klein, 1896a; cf. Loria's letter to Klein, 22 July 1895, Niedersächsische Staats-und Universitätsbibliothek, Göttingen (hereafter UBG): *F. Klein* 10, 870.

[6] Cf. for example Klein to Brioschi, 30 March 1878, Casorati to Brioschi, 4 November 1878, in *Francesco Brioschi e il suo tempo (1824–1897)* (2000). Milan: Franco Angeli, II *Inventari*, pp. 160 and 316.

[7] Cf. Cremona to Fano, 21 March 1899, Veronese to Fano, 21 March 1899, in *Fondo Fano*, Biblioteca matematica "G. Peano," Turin; Enriques to Castelnuovo, 17 March 1899 and 28 March 1899, in Bottazzini, Conte, & Gario, 1996, pp. 402 and 404.

[8] Segre to Klein, 48 letters, UBG, *F. Klein* 11; Fano to Klein, 8 letters, UBG, *F. Klein* 9 and 22; Klein to Fano, 1 letter, *Fondo Fano,* Biblioteca matematica "G. Peano," Torino; Loria to Klein, 2 letters, UBG, *F. Klein* 10; Enriques to Klein, 5 letters, UBG, *F. Klein* 4A, 8, 34, and 51; Klein to Enriques, 1 letter, UBG, *F. Klein* 51; Castelnuovo to Klein, 2 letters, UBG, *F. Klein* 51; Klein to Castelnuovo 1 letter, Gario & Palleschi, 1998, CD-ROM 2.

[9] This notebook is untitled and undated; the square brackets indicate that the title used was assigned by me.

[10] This and successive page numbers refer to the notebook [*Appunti relativi alle lezioni tenute per la Scuola di Magistero*], in Giacardi, 2002.

[11] Cf. *Atti del IV Congresso internazionale dei matematici (Roma 6–11 aprile 1908)* (1909). Roma: R. Accademia dei Lincei, 3 vols., I, pp. 45–46, and III, p. 371 ff.

[12] Cf. *Ginnasio—Liceo Moderno. Orario—Istruzioni—Programmi*, 1913, in Giacardi, 2007–2009.

[13] Castelnuovo, 1919, p. 5. On the impact of the modern *liceo* on Italian schools, see Giacardi, 2009.

[14] Enriques is referring to hands, by means of which man can measure objects.

[15] Cf. for example Enriques, 1898, *Introduzione*, pp. 3–4; and Enriques, 1906, Cap. IV.

[16] Enriques to Castelnuovo, 28 March 1899, in Bottazzini, Conte, & Gario 1996, p. 404.

[17] Enriques to Castelnuovo, 24 October 1903, in Bottazzini, Conte, & Gario 1996, p. 536.

[18]Cf. for example Enriques, F., 1924.

[19] From 1921 to 1934 Enriques shared the direction with Giulio Lazzeri; after September 1938, because of the race laws, he is no longer named as either director or author. In 1946 he once again appears as director with Oscar Chisini.

[20] Enriques, F., & Amaldi, U., 1903, *Elementi di geometria, ad uso delle scuole secondarie superiori*, Bologna: Zanichelli. Cf. also 1992 edition with a preface by Giorgio Israel (Israel, 1992).

[21] Enriques, Amaldi, 1903, p. 27. For the other aspects of this textbook cf. Giacardi, 2003b.

[22] Cf. Enriques, 1921a, and Enriques, 1921b.

[23] For further details cf. Giacardi, 2006.

[24] Cf. for example the "Introduzione" in Castelnuovo, Emma, 1963.

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